Quantum Teleportation in One-Dimensional Quantum Dots System

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Abstract

We present a model of quantum teleportation protocol based on one-dimensional quantum dots system. Three quantum dots with three electrons are used to perform teleportation, the unknown qubit is encoded using one electron spin on quantum dot A, the other two dots B and C are coupled to form a mixed space-spin entangled state. By choosing the Hamiltonian for the mixed space-spin entangled system, we can filter the space (spin) entanglement to obtain pure spin (space) entanglement and after a Bell measurement, the unknown qubit is transfered to quantum dot B. Selecting an appropriate Hamiltonian for the quantum gate allows the spin-based information to be transformed into a charge-based information. The possibility of generalizing this model to N-electrons is discussed.

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Quantum teleportation is a technique for transferring quantum states from one place to another. A concise description of the protocol for teleporting a qubit can be described as follows[1]: The sender Alice has a source qubit which she wants to send to Bob and share with Bob an entangled pair, an Einstein-Podolsky-Rosen (EPR) pair[2]. Alice does a Bell measurement on the source qubit and her half of the EPR pair, projecting the target qubit hold by Bob into a state being the same as the original state of the source qubit up to a unitary transformation. Then Bob rotate the target qubit into the original state of the source qubit based on the two bits of classical information from Alice. The details of the protocol are shown in Fig.1: Alice performs a controlled-NOT (C-NOT) operation on her two qubits, using the source qubit as the control line. Then perform a Hadamard transformation on the source qubit. Alice then performs a measurement on her two qubits. After the measurement, Alice sends Bob two bits of classical information about the result of her measurement, which is used by Bob to rotate the target qubit into the correct state. Quantum teleportation using pairs of entangled photons[3,4,5,6,7,8] and atoms[9,10] have been demonstrated experimentally. There are also schemes suggesting using electrons to perform quantum teleportation[11,12,13].

In this paper, we study the quantum entanglement in an array of quantum dots and propose a scheme to perform quantum teleportation in this system. We show that the space entanglement and spin entanglement contained in the quantum dots system modeled by the one-dimensional Hubbard Hamiltonian can be applied selectively to perform quantum teleportation.

Quantum entanglement is one of the most important concepts in quantum information theory and quantum computation. It is key to the implementation of quantum information processing technology. It has been realized that quantum entanglement can be used as a controllable physical resource[14]. Theoretically, finding a measure for the quantum entanglement is an important issue. For the fermion system, we choose Zanardi's measure[15], which is given in Fock space as the von Neumann entropy.

Quantum dots system is one of the proposals for building a quantum computer[17,18]. To describe the quantum dots, a simple approximation is to regard each dot as having one valence orbital, the electron occupation could be $|0>$, $|\uparrow>$, $|\downarrow>$ and $|\uparrow\downarrow>$, with other electrons treated as core electrons[19]. The valence electron can tunnel from a given dot to its nearest neighbor obeying the Pauli principle and thereby two dots can be coupled together,

this is the electron hopping effect. Another effect needs to be considered is the on-site electron-electron repulsion. A theoretical description of an array of quantum dots can be modeled by the one-dimensional Hubbard Hamiltonian:

$$H = -t \sum_{<ij>,\sigma} c^{+}_{i\sigma}\, c_{j\sigma} + U \sum_{i} n_{i\uparrow}\, n_{i\downarrow} \tag{1}$$

where t stands for the electron hopping parameter, U is the Coulomb repulsion parameter for electrons on the same site, i and j are the neighboring site numbers, $c^{+}_{i\sigma}$ and $c_{j\sigma}$ are the creation and annihilation operators.

Entanglement using Zanardi's measure can be formulated as the von Neumann entropy given by

$$E_j = -Tr(\rho_j log_2 \rho_j) \tag{2}$$

$$\rho_j = Tr_j(|\Psi><\Psi|) \tag{3}$$

where Tr_j denotes the trace over all but the jth site and $|\Psi>$ is the antisymmetric wave function of the fermion system. Hence E_j actually describes the entanglement of the jth site with the remaining sites.

In the Hubbard model, the electron occupation of each site has four possibilities, there are four possible local states at each site, $|\nu>_j = |0>_j, |\uparrow>_j, |\downarrow>_j, |\uparrow\downarrow>_j$. The entanglement of the jth site with the other sites is given by[20]

$$E_j = -zLog_2 z - u^{+}Log_2 u^{+} - u^{-}Log_2 u^{-} - wLog_2 w \tag{4}$$

where,

$$\rho_j = z|0><0| + u^{+}|\uparrow><\uparrow| + u^{-}|\downarrow><\downarrow| + w|\uparrow\downarrow><\uparrow\downarrow| \tag{5}$$

$$w = <n_{j\uparrow} n_{j\downarrow}> = Tr(n_{j\uparrow} n_{j\downarrow} \rho_j) \tag{6}$$

$$u^{+} = <n_{j\uparrow}> - w, \qquad u^{-} = <n_{j\downarrow}> - w \tag{7}$$

$$z = 1 - u^{+} - u^{-} - w = 1 - <n_{j\uparrow}> - <n_{j\downarrow}> + w \tag{8}$$

The Hubbard Hamiltonian can be rescaled to have only one parameter U/t.

For the one-dimensional Hubbard model with half-filled electrons, we have $<n_\uparrow> = <n_\downarrow> = \frac{1}{2}$, $u^{+} = u^{-} = \frac{1}{2} - w$, and the local entanglement is given by

$$E_j = -2wlog_2 w - 2(\frac{1}{2} - w)log_2(\frac{1}{2} - w) \tag{9}$$

For each site the entanglement is the same. Consider the particle-hole symmetry of the model, we can see that $w(-U) = \frac{1}{2} - w(U)$, so the local entanglement is an even function of U. As shown in Fig. 2, the minimum of the entanglement is 1 as $U \to \pm\infty$. As $U \to +\infty$ all the sites are singly occupied the only difference is the spin of the electrons on each site, which can be referred as the spin entanglement. As $U \to -\infty$, all the sites are either doubly occupied or empty, which is referred as the space entanglement. The maximum entanglement is 2 at $U = 0$, which is the sum of the spin and space entanglement of the system. In Fig. 2 we show the entanglement for two sites and two electrons, they qualitatively agree with that of the Bethe ansatz solution for an array of sites[20].

Gittings and Fisher[16] showed that the entanglement in this system can be used in quantum teleportation. However, in their scheme both the charge and spin of the system are used to construct the unitary transformation. In this paper, we propose a different scheme to perform quantum teleportation. For two half-filled coupled quantum dots, under the conservation of the total number of electrons $N = 2$ and the total electron spin $S = 0$, a quantum entanglement of 2, two ebits can be produced according to Zanardi's measure. Let us describe the teleportation scheme using three cites, A, B and C. Suppose the qubit $\alpha| \uparrow> +\beta| \downarrow>$ will be teleported from site A, Alice, to site B, Bob, where the two sites B and C are in an entangled state,

$$|\Psi >= \frac{1}{\sqrt{2}}(c^+_{C\uparrow} + c^+_{B\uparrow}) \frac{1}{\sqrt{2}}(c^+_{C\downarrow} + c^+_{B\downarrow})|0> . \tag{10}$$

A spin-up electron and a spin-down electron are in a delocalized state on sites C and B. In the occupation number basis $| \ n_{C\uparrow} \ n_{C\downarrow} \ n_{B\uparrow} \ n_{B\downarrow} >$, the state of the system can be written as:

$$|\Psi >= \frac{1}{\sqrt{2}}(c^+_{C\uparrow} + c^+_{B\uparrow}) \frac{1}{\sqrt{2}}(c^+_{C\downarrow} + c^+_{B\downarrow})|0 >= \frac{1}{2}(|0011 > +|1100 > +|1001 > +|0110 >). \tag{11}$$

From the state described by Eq. (10) we can see that in the basis of $| \ n_{C\uparrow} \ n_{C\downarrow} >$, there are four possible states: $|00 >$, $|11 >$, $|10 >$, $|01 >$. Corresponding to each of the states on site C, the states on site B are: $|11 >$, $|00 >$, $|01 >$, $|10 >$ in the occupation number basis

$| n_{B\uparrow}\, n_{B\downarrow} >$. Under the restriction of the conservation of total number of electrons and total spin of the system, two ebits can be obtained, one is in the spatial degree of freedom, and the other is in the spin degree of freedom. In the basis of $| n_{C\uparrow}\, n_{C\downarrow}\, n_{B\uparrow}\, n_{B\downarrow} >$, the two ebits are:

$$\beta_0 = \frac{1}{\sqrt{2}}(|1100> + |0011>), \qquad \beta_1 = \frac{1}{\sqrt{2}}(|1001> + |0110>) \tag{12}$$

These two ebits can be used in quantum teleportation. The C-NOT operation in the occupation number basis $| n_{A\uparrow}\, n_{A\downarrow}\, n_{C\uparrow}\, n_{C\downarrow} >$ is given by:

$$|1000> \leftrightarrow |1011>, |1010> \leftrightarrow |1001>, |01\, n_{C\uparrow}\, n_{C\downarrow}> \leftrightarrow |01\, n_{C\uparrow}\, n_{C\downarrow}> \tag{13}$$

For the ebit β_0, in the quantum teleportation process, in basis $| n_{A\uparrow}\, n_{A\downarrow} > | n_{C\uparrow}\, n_{C\downarrow}\, n_{B\uparrow}\, n_{B\downarrow} >$, as shown in Fig. 1, we have:

$$|\Psi_0> = (\alpha|10> + \beta|01>)\frac{1}{2}(|1100> + |0011> + |1001> + |0110>) \tag{14}$$

$$|\Psi_1> = \alpha|10> \frac{1}{\sqrt{2}}(|0000> + |1111>) + \beta|01> \frac{1}{\sqrt{2}}(|1100> + |0011>) \tag{15}$$

$$|\Psi_2> = \alpha(|10> + |01>)\frac{1}{2}(|0000> + |1111>) + \beta(|10> - |01>)\frac{1}{2}(|1100> + |0011>) \tag{16}$$

When Alice does the measurement M_1 and M_2, the following results can be obtained:

$$\begin{array}{ll} |M_1 M_2> & | n_{B\uparrow}\, n_{B\downarrow}> \\ |1011> & \alpha|11> + \beta|00> \\ |1000> & \alpha|00> + \beta|11> \\ |0111> & \alpha|11> - \beta|00> \\ |0100> & \alpha|00> - \beta|11> \end{array} \tag{17}$$

Then after doing a unitary transformation using double electron occupation and zero electron occupation as basis, the source qubit can be obtain on site B. For this system the Hamiltonian to perform the C-NOT operation is given by:

$$
\begin{aligned}
H &= |10>_{A\ A}<10|(|11>_{C\ C}<00|+|00>_{C\ C}<11|)+ \\
&|01>_{A\ A}<01|(|11>_{C\ C}<11|+|00>_{C\ C}<00|) \\
&= \frac{1}{2}(\sigma_{Z,A}+1)(\ c^+_{C\uparrow}\ c^+_{C\downarrow}+\ c_{C\uparrow}\ c_{C\downarrow})+ \\
&\frac{1}{2}(1-\sigma_{Z,A})(\ c^+_{C\uparrow}\ c^+_{C\downarrow}\ c_{C\uparrow}\ c_{C\downarrow}+\ c_{C\uparrow}\ c_{C\downarrow}\ c^+_{C\uparrow}\ c^+_{C\downarrow}),
\end{aligned}
\tag{18}
$$

where $\sigma_{Z,A}$ is the Pauli matrix. We can see that by using this Hamiltonian, the spin entanglement of the system is filtered, the space entanglement is used in the process. An important point is that the original state we try to teleport is in a superposition state of electron spin up and spin down. However, after the teleportation process, the state we obtain on site B is a superposition state of double electron occupation and zero electron occupation. The information based on spin has been transformed to information based on charge, but the information content is not changed. It is well known that a difficult task in quantum information processing and spintronics is the measurement of a single electron spin[21], in the scheme above, we changed the quantum information from spin-based to charge-based, thus makes the measurement fairly easier. This is also important in quantum computation based on electron spin since the readout can be easily measured.

For another ebit β_1, in the quantum teleportation process, in basis $|\ n_{A\uparrow}\ n_{A\downarrow}\ >\ |\ n_{C\uparrow}\ n_{C\downarrow}\ n_{B\uparrow}\ n_{B\downarrow}\ >$, we have:

$$
|\Psi_0>=(\alpha|10>+\beta|01>)\frac{1}{2}(1100>+|0011>+|1001>+|0110>) \tag{19}
$$

$$
|\Psi_1>=\alpha|10>\frac{1}{\sqrt{2}}(|0101>+|1010>)+\beta|01>\frac{1}{\sqrt{2}}(|1001>+|0110>) \tag{20}
$$

$$
|\Psi_2>=\alpha(|10>+|01>)\frac{1}{2}(|0101>+|1010>+\beta(|10>-|01>)\frac{1}{2}(|1001>+|0110>) \tag{21}
$$

When Alice does the measurement M_1 and M_2, the following results can be obtained:

$\|M_1M_2>$	$\|\ n_{B\uparrow}\ n_{B\downarrow}\ >$
$\|1001>$	$\alpha\|01>+\beta\|10>$
$\|1010>$	$\alpha\|10>+\beta\|01>$

$$
\begin{aligned}
&|0101> && \alpha|01> -\beta|10> \\
&|0110> && \alpha|10> -\beta|01>
\end{aligned} \tag{22}
$$

For this system the Hamiltonian to perform the C-NOT operation is:

$$
\begin{aligned}
H \;=\; & |10>_{A\ A} < 10|(|10>_{C\ C} < 01| + |01>_{C\ C} < 10|) + \\
& |01>_{A\ A} < 01|(|01>_{C\ C} < 01| + |10>_{C\ C} < 10|) \\
& = \frac{1}{2}(\sigma_{Z,A} + 1)(\ c^{+}_{C\uparrow}\ c_{C\downarrow} + \ c^{+}_{C\downarrow}\ c_{C\uparrow}) \\
& + \frac{1}{2}(1 - \sigma_{Z,A})(\ c^{+}_{C\uparrow}\ c_{C\uparrow}\ c_{C\downarrow}\ c^{+}_{C\downarrow} + \ c^{+}_{C\downarrow}\ c_{C\downarrow}\ c_{C\uparrow}\ c^{+}_{C\uparrow})
\end{aligned} \tag{23}
$$

Then after doing a unitary transformation using the electron spin up and spin down as basis the source qubit can be recovered on site B. By using this Hamiltonian for the C-NOT operation the space entanglement of the system is filtered, the spin entanglement is used in the process. In the case of using β_0 or β_1 as ebits, the unitary transformation is performed in the occupation number basis of $|\ n_{B\uparrow}\ n_{B\downarrow} >$, using basis $|11>, |00>$ or $|10>, |01>$, we can select the basis separately, either charge or spin. We can also choose the Hamiltonians (one is related to the spin entanglement and the other is related to space entanglement) for the C-NOT operation, when the Hamiltonian for one ebit is chosen, the ebit corresponding to the other Hamiltonian will be filtered.

For $U \neq 0$, the state of the 2-electron 2-sites system can be described as follows:

$$
|\Psi >= a_1|1100> + a_2|0011> + b_1|1001> + b_2|0110>;\ \ a_1^2 + a_2^2 + b_1^2 + b_2^2 = 1, \tag{24}
$$

where $a_1 = a_2$, $b_1 = b_2$ because of the symmetry in the entangled pairs, such that the state can be written as:

$$
|\Psi >= a\beta_0 + b\beta_1;\ \ a^2 + b^2 = 1. \tag{25}
$$

If $U > 0$, the contribution of the spin entanglement to the total entanglement is greater than that of the space entanglement. The probability of getting the ebit $|\beta_1>$ increases as U becomes larger. If $U < 0$, the contribution of the space entanglement to the total entanglement becomes greater than that of the spin entanglement, the probability of getting

the ebit $|\beta_0>$ increases as U becomes more negative. In the limit of U goes to $\pm\infty$, only spin entanglement or space entanglement will exist. This might be related to the spin charge separation in the Hubbard model[22]. In a previous study[23], we showed that the maximum entanglement can be reached at $U > 0$ by introducing asymmetric electron hopping impurity to the system. This is very convenient in the quantum information processing. We can control the parameter U/t to increase the probability of getting either ebit.

Here, we discussed implementing quantum teleportation in three-electron system. For more electrons and in the limit of $U \rightarrow +\infty$ there is no double occupation, the system reduced to the Heisenberge model, in the magnetic field. The neighboring spins will favor the anti-parallel configuration for the ground state. If the spin at one end is flipped, then the spins on the whole chain will be flipped accordingly due to the spin-spin correlation. Such that the spins at the two ends of the chain are entangled, a spin entanglement, this can be used for quantum teleportation, the information can be transfered through the chain. For $U \neq +\infty$, for the N-sites N-electron system with $S = 0$, the first $N-1$ sites entangled with the N-th site in the same way as that of the two-electron two-sites system: if the N-th site has 2 electrons, then the first $N-1$ sites will have $N-2$ electrons; if the N-th site has 0 electrons, then the first $N-1$ sites will have N electrons; if the N-th site has 1 spin-up electron, then the total spin of the first $N-1$ sites will be 1 spin-down; if the N-th site has the 1 spin-down electron, then the total spin of the first $N-1$ sites will be 1 spin-up. So the same procedure discussed above can be used for quantum teleportation but the new system with N-electrons is much more complicated than the previous three electron system. Moreover, Alice needs to control the first $N-1$ sites and the source qubit. This situation is different from the spin chain, this correlation can not be transfered from one end to the other.

In summary, we have studied the entanglement of an array of quantum dots modeled by the one-dimensional Hubbard Hamiltonian and its application in quantum teleportation. The entanglement in this system is a mixture of space and spin entanglement. The application of such entanglement in quantum teleportation process has been discussed, by applying different Hamiltonians for the C-NOT operation, we can separate the ebit based on space entanglement or spin entanglement and apply it in quantum teleportation process. It turns out that if we use the ebit of the space entanglement, we can transform the spin-based quantum information to the charge-based quantum information making the measurement

fairly easy.

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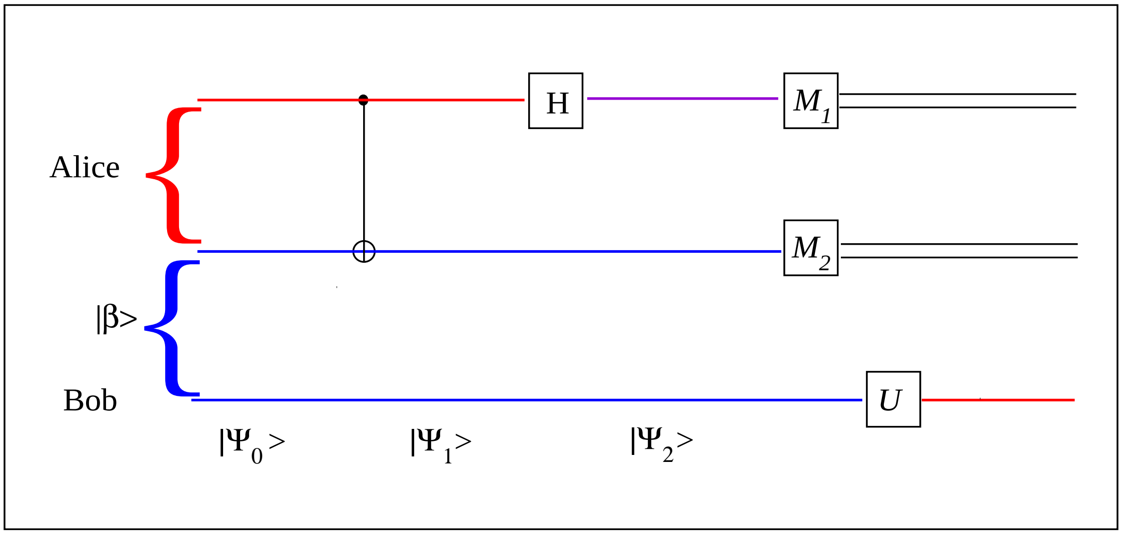

FIG. 1: Quantum circuit for teleporting a qubit. The two top lines represent Alice's system, while the bottom line is Bob's system. H represents a Hadmard transformation, M_1 and M_2 represent the measurement on the two top lines.

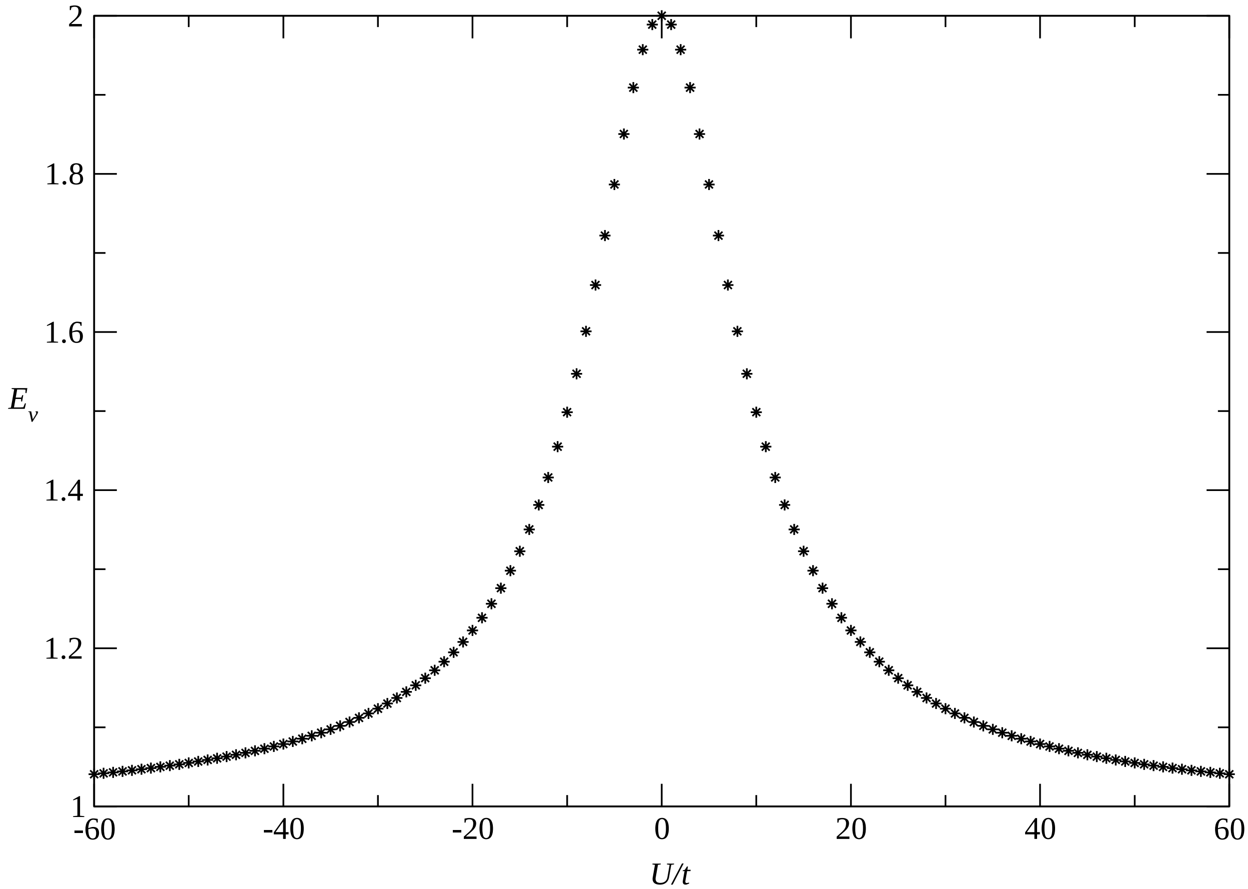

FIG. 2: Local entanglement given by the von Neumann entropy E_v versus U/t.